

India VIX Futures Theoretical Pricing Model

For computing the fair value of stock index futures, the cost-of carry relationship between the futures and the underlying stock index is used. However, since there is no carry between India VIX futures and India VIX, the fair value of India VIX futures cannot be derived by a similar relationship. Instead the fair value is derived by pricing the expected forward 30-day variance from the expiry of the India VIX futures contract. The expected forward 30-day variance is derived from the term structure of average variance rate.

The fair value of the India VIX futures contract (F_T) expiring T calendar days from the current date is calculated using the following formula¹:

$$F_T = 100 * \sqrt{((T + 30) * V_{T+30} - T * V_T) * 365/30} \quad :: \text{Equation (1)}$$

where, V_{T+30} is the average variance rate over the next 'T+30' calendar days and V_T is the average variance rate over the next 'T' calendar days. V_{T+30} and V_T are derived from the term structure of average variance rate. For constructing the term structure, the GARCH (1, 1) model is used which is explained below:

According to the GARCH (1, 1) model, the expected daily variance σ_n^2 on day 'n' is given by:

$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2 \quad :: \text{Equation (2)}$$

where, V_L is the long-term daily variance, u_{n-1}^2 is the squared continuously compounded return on day 'n-1' and α , β & γ are weights that sum to unity. Using the GARCH (1, 1) model, the term structure² of average variance rate (V_t) over the next 't' calendar days is given as:

$$V_t = V_L + \frac{(1-e^{-at})}{at} (V_0 - V_L) \quad :: \text{Equation (3)}$$

² John C. Hull, "Options, Futures and other Derivatives"

where, V_0 is the instantaneous daily variance and 'a' is the speed with which daily variance reverts to its long-term mean (mean-reversion speed).

India VIX is a measure of the annualized expected volatility over the next 30 calendar days. Hence, the relationship between India VIX and the average variance rate (V_{30}) between today and 30 calendar days from now is given as:

$$V_{30} = \left(\frac{(\text{India VIX}/100)^2}{365} \right) \quad :: \text{Equation (4)}$$

The long-term daily variance (V_L) is computed by taking the simple average of average variance rates (V_{30}) computed using India VIX values over the past 90 trading days on a rolling basis excluding current day. The mean-reversion speed (a) is computed on a rolling basis by minimizing the sum of the estimation errors over the past 90 trading days excluding the current day. The error term is defined as:

$$\text{error term} = \frac{(\text{model future price} - \text{actual future price})^2}{\text{model future price}} \quad :: \text{Equation (5)}$$

Using equation (3) with $t = 30$ and rearranging the terms, we get the formula for V_0 as:

$$V_0 = V_L + \frac{30a}{(1 - e^{-30a})} * (V_{30} - V_L) \quad :: \text{Equation (6)}$$

Once all the parameters (viz. V_L , V_0 & 'a') have been estimated, the term structure for average variance rate is constructed using equation (3). The values of V_T and V_{T+30} are computed using the term structure. These values are used to compute the fair value of the India VIX futures contract (FT) using equation (1).